Matrices

$$A = [a_{ij}] = [A]$$

row →

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

column

order

row × column

2 × 4

↓ rows ↓ columns

entry

a_{ij}

i^{th} row

j^{th} column

Equality

$[A] = [B]$ if they have same order and equal corresponding values

$$\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad a=2 \quad b=-3 \quad c=0 \quad d=1$$

Add/subtracting

$$\begin{bmatrix} 3 & 1 & 6 \\ 2 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 1 & -1 \\ 6 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ 8 & 4 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 \\ 8 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 3 \end{bmatrix} = \emptyset$$

Scalar Multiplication

$$6\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 12 & 6 \\ 0 & 18 \end{bmatrix}$$

Multplication

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 2\cdot 0 + 1\cdot 4 & 2\cdot 2 + 1\cdot 3 \\ 3\cdot 0 + 4\cdot 4 & 3\cdot 2 + 4\cdot 3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 16 & 18 \end{bmatrix}$$

Problems

① What number is in ⓐ 3rd row, 2nd column? ⓑ 2nd row 3rd column

$$\begin{bmatrix} 1 & 3 & 6 \\ 27 & 0 & 4 \\ 3 & 9 & 2 \end{bmatrix}$$

② $\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 9 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 6 \\ 6 & 8 \end{bmatrix}$

③ $\begin{bmatrix} 5 & 8 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ -2 & -4 \end{bmatrix}$

④ $\begin{bmatrix} 2 & 3 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \emptyset$

⑤ $\begin{bmatrix} 8 & 2 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 \\ 1 & -5 \end{bmatrix} = \begin{bmatrix} -14 & -10 \\ 4 & -20 \end{bmatrix}$

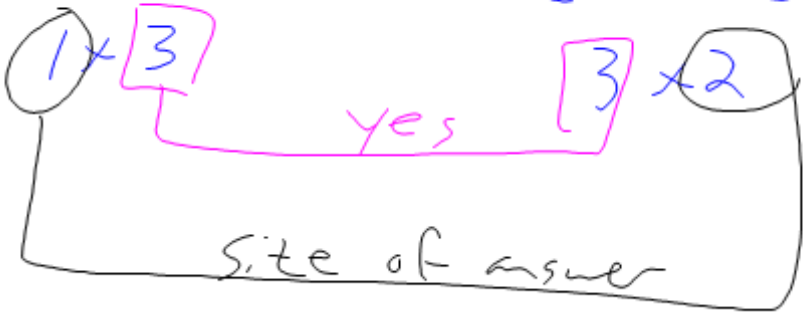
⑥ $\begin{bmatrix} 3 & 0 \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 6 \\ 14 & 4 \end{bmatrix}$

⑦ $\begin{bmatrix} 3 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 3 & 0 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 11 & 15 \end{bmatrix}$

⑧ $\begin{bmatrix} 2 & 4 \\ 3 & 0 \\ -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 & -1 \end{bmatrix} = \emptyset$

$3 \times \textcircled{2} \cdot \textcircled{1} \times 3$

$$\begin{bmatrix} 3 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 3 & 0 \\ -2 & -3 \end{bmatrix}$$



$$= \begin{bmatrix} 3\cdot 2 + 1\cdot 3 + -1\cdot -2 & 3\cdot 4 + 1\cdot 0 + -1\cdot -3 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 15 \end{bmatrix}$$

16 | 7.3

$$\begin{aligned} 4x + y - 3z &= 11 \\ 2x - 3y + 2z &= 9 \\ x + y + z &= -3 \end{aligned}$$

augmented matrix

$$\left[\begin{array}{ccc|c} 4 & 1 & -3 & 11 \\ 2 & -3 & 2 & 9 \\ 1 & 1 & 1 & -3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 2 & -3 & 2 & 9 \\ 4 & 1 & -3 & 11 \end{array}\right]$$ switched $R_1 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -5 & 0 & 15 \\ 4 & 1 & -3 & 11 \end{array}\right] \quad -2R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -5 & 0 & 15 \\ 0 & -3 & -7 & 23 \end{array}\right] \quad -4R_1 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & 1 & 0 & -3 \\ 0 & -3 & -7 & 23 \end{array}\right] \quad R_2 \div -5$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & -7 & 14 \end{array}\right] \quad -3 \cdot R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$(2, -3, -2)$$

$$4x + y - 3z = 11$$
$$2x - 3y + 2z = 9$$
$$x + y + z = -3$$

$$\Rightarrow \begin{bmatrix} 4 & 1 & -3 \\ 2 & -3 & 2 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ -3 \end{bmatrix}$$

$$[A]^{-1} \cdot [A] \cdot [X] = [B] \quad \text{(multiply by } [A]^{-1}\text{)}$$

Coefficient matrix

Variable matrix

Answer matrix

$$\frac{1}{A} \cdot AX = B \cdot \frac{1}{A}$$

$$X = \frac{1}{A} \cdot B$$

$$[X] = [A]^{-1} \cdot [B]$$

$$[X] = \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix}$$

7.3 #17-19, 38

7.4 #2-12 (even), 55, 59 (easy)

} Do by hand, check on calc.